In memory of Costas Kounnas

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CNRS and Ecole Polytechnique

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String Pheno 2022, Liverpool, 4 - 8 July, 2022

■ Our friend and colleague Costas Kounnas passed over in January, just before his 70th birthday.

■ Strange feeling to present very few aspects of his personality and career because he was extremely lively, had a strong personality + outstanding physicist.

■ I have never seen so many exchanges of emails, pictures, memories.

■ He was **enjoying people!**

He always liked having people around, talking with them, inviting them and cooking for them.

His style was not to make fancy, sophisticated food. He liked traditional things, to share friendly.

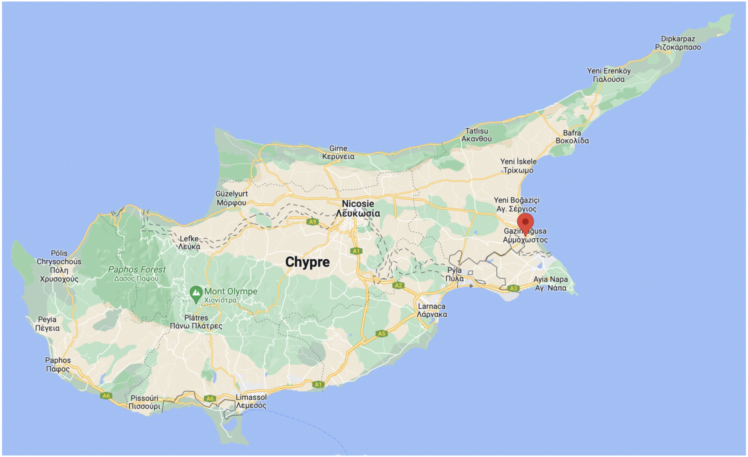
■ He had a lot of friends, in physics and from other origins, and he was always introducing them to each other and wanted to create links between them.

■ He was very faithful in friendship and helping them when possible.

■ He was never shy and was **enjoying life** (and smoking, eating and also drinking...)

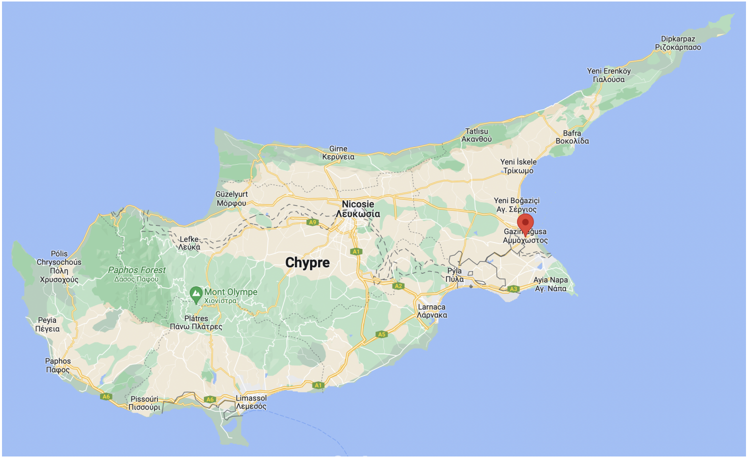
- He said he is a "citizen of the world."
  - This may illustrate his character, which does not like limitations.
  - Maybe related to his personal history:

- He was **born in Cyprus in 1952, in a town called Famagusta**



• He was always talking about this town as a **lost paradise.** With the most beautiful beaches of the Island.

- However, in 1974 the Turks invaded the island and started to occupy the northern part of it.



Since then the country is divided in 2: The occupied part and the part that is member of the 27 countries of EEC

- Famagusta is just at the border:

• The town was taken in 2 days and all its inhabitants left, thinking that they would come back soon after everything gets fixed.

• However, things did not happen like that. Access to the town has been forbidden by the Turks. Only this town was abandoned to itself, surrounded by barbed wire since 1974.

• Since then, cactus and plants are growing up in the houses. They can be seen when you walk along the barbed wire.

■ **Costas was 22 years old.** He was doing his military service.

■ Then he came to Paris to do his Master + thesis with John Iliopoulos on QCD.

• He would have probably come to Paris in any case, to become a "citizen of the world."
However, since he was kicked out of his lost paradise of Famagusta, maybe living his island was more difficult than expected.

• In fact, he kept very strong links with his island. Going back and forth. He was for many years the official representative of the Cypriot community in France. And he also never asked for the French nationality. Maybe to remain faithful to its origins?

Costas and science

■ Costas was an **outstanding physicist.** Not only because $\sim$200 publications and $>$90 collaborators

- His own way of thinking: "I am not a follower"

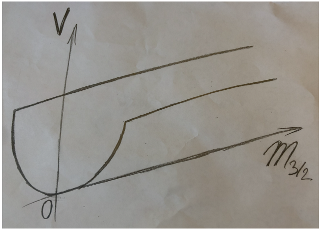
- He introduced several new ideas and technics over the years, which opened totally new directions. Examples:

■ In the early days of supergravity, supersymmetric vacua were known in Minkowski spacetime.

It was believed that introducing some kind of breaking of supersymmetry would always introduce a non-trivial (large) cosmological constant.

- **In 1983,** Cremmer, Ferrara, C.K., Nanopoulos: This is not true!

You can generate a potential ≥ 0, with vanishing minima



The scale of susy breaking $m_{3/2}$ is a flat direction: **No-scale models**

- Attempts for constructing models with **hierarchy of scales.**

■ **In 1988,** Ferrara, C.K, Porrati constructed string theory models realizing the spontaneous breaking of susy in Minkowski space at tree level:

• Explicit worldsheet CFT.

• Spacetime d.o.f. in the each supermultiplet have different boundary conditions along compact directions. String version of the Scherk-Schwarz mechanism, '79.

■ **In 1990,** C.K and Rostand showed how to switch on **finite temperature in string theory:**

• Implement periodic/antiperiodic boundary conditions for bosons/fermions along the Euclidean time, while preserving modular invariance.

■ **In 1986,** Antoniadis, Bachas, C.K showed how to construct consistent string theory models in 4 dimensions.

• Not true to say that superstring theory leaves in 10D. Only far in moduli space.

• At special points, the 2D CFT can be realized in terms of free fermions on the worldsheet $\Longrightarrow$ **Fermionic constructions**

■ And so many other fields!!!

- String cosmology (Antoniadis, Bachas, Lüst, Kiritsis, Cornalba, Costa, Partouche, Toumbas, Florakis,...)
- MSSM, GUT (Ferrara, Zwirner, Pavel, Nanoupoulos, Lahanas, Nanopoulos, Quiros,...)
- QCD (Baulieu, Floratos, Lacaze,...)
- Threshold corrections to string couplings (Petropoulos, Rizos, Kiritsis,...)
- Renormalization group in string theory (Kiritsis,...)
- String model building using fermionic constructions (Faraggi, Rizos, Nooij, Assel, Christodoulides,...)
- Hagedorn phase transition (Antoniadis, Derendinger, Axenides, S.D. Ellis, Angelantonj, Partouche, Toumbas,...)
- String dualities (Petropoulos, Israël,...)
- Flux compactifications (Derendinger, Petropoulos, Zwirner,...)
- Moduli stabilization (Partouche, Toumbas, Estes,...)
- R^2-theory (Lüst, Toumbas, Kehagias, Riotto, Álvarez-Gaumé,...)

Memorial day organized at the

CORFU SUMMER INSTITUTE

4 September, 2022

http://physics.ntua.gr/corfu2022/ko.html

Wavefunction of the universe: Invariance under field redefinitions

Based on

H.P., N. Toumbas, B. de Vaulchier, Nucl. Phys. B 973 (2021), 115600

A. Kehagias, H.P., N. Toumbas JHEP 12 (2021), 165

Introduction

■ In Quantum Mechanics, wavefunctions $\Longrightarrow$ probabilities

- Schrödinger equation
- Path integral

■ In Quantum Gravity, "wavefunction of the universe"

$\Longrightarrow$ probabilities favoring realistic aspects of the Universe?

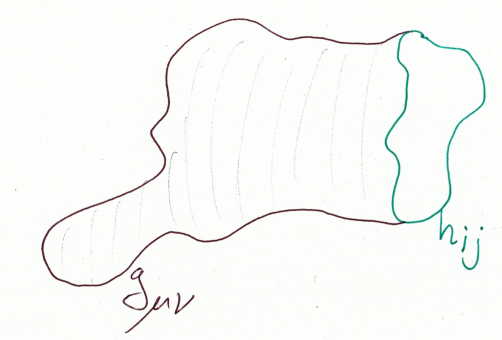
- Wheeler DeWitt equation ['62]
- Hartle-Hawking proposal for spatially closed universes with cosmological constant $\Lambda > 0$ ['83]

■ Both have **ambiguities that we are going to lift.**

■ Hartle-Hawking Euclidean path integral

$$\Psi[h_{ij}] = \int \frac{\mathcal{D}g}{\mathrm{Vol(Diff)}}\, e^{-\frac{1}{\hbar} S_{\mathrm{E}}[g]}$$

where the sum is over all **compact four-manifolds of Euclidean metric $g_{\mu\nu}$ that have a single 3D boundary of metric h_{ij}**

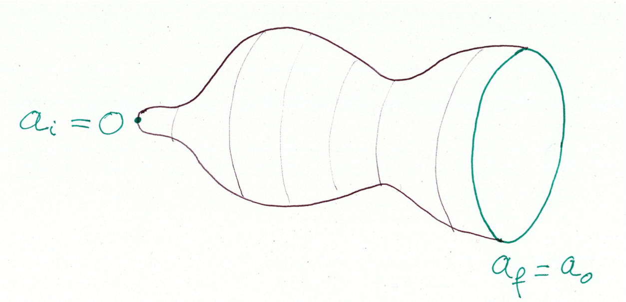


■ Homogeneous and isotropic: Space is a 3-sphere

$$\mathrm{d}s^2 = -N(t)^2 \mathrm{d}t^2 + a(t)^2 \, \mathrm{d}\Omega_3^2$$

$$\Psi(a_0) = \int_{\substack{a_\mathrm{i}=0 \\ a_\mathrm{f}=a_0}} \frac{\mathcal{D}N \, \mathcal{D}a}{\mathrm{Vol(Diff)}} \; e^{-\frac{1}{\hbar} S_\mathrm{E}[N,a]}$$

where Diff = Euclidean-time reparametrizations



Gauge fixing of Euclidean-time reparametrizations

■ The Euclidean action is

$$S_{\mathrm{E}} = 6\pi \int_{x^0_{\mathrm{Ei}}}^{x^0_{\mathrm{Ef}}} \mathrm{d}x^0_{\mathrm{E}} \sqrt{g_{00}} \left[a\, g^{00} \left(\frac{\mathrm{d}a}{\mathrm{d}x^0_{\mathrm{E}}} \right)^2 + a - \frac{\Lambda}{3}\, a^3 \right]$$

■ It describes a **non-linear $\boldsymbol{\sigma}$-model, like in String Theory:**

- There is a **line segment** $[\boldsymbol{x^0_{\mathrm{Ei}}}, \boldsymbol{x^0_{\mathrm{Ef}}}]$ of metric $g_{00} \equiv N^2$

- **The target space is $\mathbb{R}_+$ parametrized by the scale factor $\boldsymbol{a}$,** with metric $G_{aa} = a$.

■ Let us concentrate on the line-segment of Euclidean time:

All metrics g_{00} are not equivalent up to a change of coordinate, since the proper length ℓ of a line segment is invariant under a change of coordinate.

$\Longrightarrow$ To fix a gauge, we choose a metric $\hat{g}_{00}[\ell]$ in each equivalence class ℓ, which is a modulus

$$\int \frac{\mathcal{D}N}{\mathrm{Vol(Diff)}} = \int_0^{+\infty} \mathrm{d}\ell \; \cancel{\int_{\mathrm{Diff}} \frac{\mathcal{D}\xi}{\mathrm{Vol(Diff)}}} \; \Delta_{\mathrm{FP}}[\hat{g}_{00}[\ell]]$$

■ **Fadeev-Popov determinant**

$$\Delta_{\mathrm{FP}}[\hat{g}_{00}[\ell]] = 1$$

Path integral over the scale factor

- Gauge $\hat{g}_{00}[\ell] = \ell^2$ defined on $[0, 1]$

$$\Psi(a_0) = \int_0^{+\infty} \mathrm{d}\ell \int_{\substack{a(0)=0 \\ a(1)=a_0}} \mathcal{D}a \; e^{-\frac{1}{\hbar} S_{\mathrm{E}}[\ell, a]}$$

where the action

$$S_{\mathrm{E}}[\ell, a] = 6\pi \int_0^1 \mathrm{d}\tau \left[\frac{a}{\ell} \left(\frac{\mathrm{d}a}{\mathrm{d}\tau} \right)^2 + \ell \, V(a) \right], \qquad V(a) = a - \frac{\Lambda}{3} a^3$$

- It is not quadratic $\Longrightarrow$ **semi-classical approximation**

$$\Psi(a_0) = \sum_{\epsilon = \pm 1} \frac{1}{\sqrt{\epsilon}} \frac{\exp\left[\epsilon \, \frac{12\pi^2}{\hbar \Lambda} \left(1 - \frac{\Lambda}{3} a_0^2 \right)^{\frac{3}{2}} \right]}{a_0^{\frac{1}{8}} \left(1 - \frac{\Lambda}{3} a_0^2 \right)^{\frac{1}{4}}} \left(1 + \mathcal{O}(\hbar) \right)$$

Field redefinitions

■ Classically, the action is invariant under redefinitions $\boldsymbol{a = A(q)}$.
At the quantum level $\boldsymbol{\mathcal{D}a \neq \mathcal{D}q}$ **due to a Jacobian**

$$\widetilde{\Psi}(q_0) = \int_0^{+\infty} \mathrm{d}\ell \int_{\substack{q(0)=q_\mathrm{i} \\ q(1)=q_0}} \mathcal{D}q \; e^{-\frac{1}{\hbar} S_\mathrm{E}[\ell^2, q]} \qquad \text{where} \qquad a_0 = A(q_0)\,, \quad 0 = A(q_\mathrm{i})$$

$$= \sum_{\epsilon=\pm 1} \frac{1}{\sqrt{\epsilon}} \frac{\exp\left[\epsilon\, \frac{12\pi^2}{\hbar\,\Lambda} \left(1 - \frac{\Lambda}{3} a_0^2\right)^{\frac{3}{2}}\right]}{|A'(q_0)|^{-\frac{1}{4}}\, a_0^{\frac{1}{8}} \left(1 - \frac{\Lambda}{3} a_0^2\right)^{\frac{1}{4}}} \left(1 + \mathcal{O}(\hbar)\right)$$

There are **infinitely many different choices of wavefunctions!**

Wheeler-DeWitt equation

■ For each prescription $\mathcal{D}q$, the **wavefunction should satisfy an equation** similar to Schrödinger in quantum mechanics:

■ **Wheeler-DeWitt equation says that the quantum Hamiltonian vanishes on all states of the Hilbert space**

■ Classically, we have for arbitrary functions $\rho_1(q), \rho_2(q)$

$$\pi_q^2 = \frac{1}{\rho_1\,\rho_2}\,\pi_q\,\rho_1\,\pi_q\,\rho_2$$

Canonical quantization $q \to q_0\,, \quad \pi_q \to -i\hbar\,\frac{\mathrm{d}}{\mathrm{d}q_0} \Longrightarrow$ **ambiguity**

$$\frac{\hbar^2}{24\pi}\,\frac{1}{AA'^2}\,\frac{1}{\rho}\,\frac{\mathrm{d}}{\mathrm{d}q_0}\Big(\rho\,\frac{\mathrm{d}\Phi}{\mathrm{d}q_0}\Big) + \Big(\hbar^2\,\omega - 6\pi V\Big)\Phi = 0$$

■ We can find ρ by solving this equation at the semi-classical level using the **WKB method**

$$\Phi(q_0) = \sum_{\epsilon=\pm 1} N_\epsilon \frac{\exp\left[\epsilon s \, \frac{12\pi^2}{\hbar\,\Lambda} \left(1 - \frac{\Lambda}{3} a_0^2\right)^{\frac{3}{2}}\right]}{\sqrt{a_0 \, \rho(q_0) |A'(q_0)|} \left(1 - \frac{\Lambda}{3} a_0^2\right)^{\frac{1}{4}}} \left(1 + \mathcal{O}(\hbar)\right)$$

Comparing with a particular wavefunction, the “no-boundary state”

$$\Longrightarrow \quad \rho(q_0) = a_0^{-\frac{3}{4}} \, |A'(q_0)|^{-\frac{3}{2}}$$

Universality at the semi-classical

■ **Different wavefunction prescriptions $\mathcal{D}q$ and Wheeler-DeWitt equations**
$\Longrightarrow$ **different quantum gravities with same classical limits?**

• To discuss probabilities, we **define an inner product in each Hilbert space.**

$$\langle \Phi_1, \Phi_2 \rangle = \int_0^{+\infty} \mathrm{d}a_0 \, \mu(a_0) \, \Phi_1(a_0)^* \, \Phi_2(a_0)$$

• Impose **Hermiticity of the Hamiltnonian**

$$\Big\langle \Phi_1, \frac{\mathcal{H}}{N} \, \Phi_2 \Big\rangle = \Big\langle \frac{\mathcal{H}}{N} \, \Phi_1, \Phi_2 \Big\rangle$$

$\Longrightarrow$ Differential equation $\Longrightarrow$ $\mu = a_0 \, \rho \, |A'|$

- The inner product

$$\langle \Phi_1, \Phi_2 \rangle = \int_0^{+\infty} \mathrm{d}a_0 \, \mu \, \Phi_1^* \, \Phi_2$$

is **independent of $\boldsymbol{\rho}$ and $\boldsymbol{A}$, *i.e.* independent of the choice of measure $\boldsymbol{\mathcal{D}q}$,** at least at the semi-classical level.

■ **If we assume this statement is exact in $\hbar$,** we can lift completely the remaining ambiguity in the Wheeler-DeWitt equation

$$\Longrightarrow \quad \omega = -\frac{1}{24\pi^2 a_0} \left[\frac{5}{16} \frac{1}{a_0^2} + \frac{1}{4} \Big(\frac{\rho'}{\rho} \Big)^2 - \frac{\rho''}{\rho} \right]$$

Recovering the classical cosmology

■ Recover FRW cosmology from the quantum wavefunction by taking $\hbar \to 0$. [A.Kehagias, H.P., N. Toumbas, '21]

■ Case where the **universe is filled with a perfect fluid** of state equation $p_m = w\rho_m$

■ **Classical probability =** duration the scale factor lies in the range $[a, a + \mathrm{d}a]$, divided by the total duration of the cosmological evolution.

This duration is finite for $w > -1/3$: Big Bang-Big Crunch.

It is not when $w < -1/3$: Bounce. We keep it not normalized.

■ $P(a) \equiv \mu|\Phi|^2 \longrightarrow P_{\mathrm{cl}}(a) \quad \text{when} \quad \hbar \to 0$

Conclusion

■ We have considered the **Hartle-Hawking wavefunction for spatially closed universes, homogeneous and isotropic.**

■ The **field redefinitions of the scale factor yield different path-integral measures, wavefunctions and Wheeler-DeWitt equations, but the quantum predictions are universal** at least semi-classically.

■ The **quantum probabilities reproduce in the $\hbar \to 0$ limit the classical cosmological evolution.**